

Quotient-3 Cordial Labelling For Cycle Related Graphs

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Abstract: Let G be a graph of order p and size q . Let $f: V(G) \rightarrow Z_4 - \{0\}$ be a function. For each $E(G)$ define $f^*: E(G) \rightarrow Z_3$ by $f^*(uv) = \left[\frac{f(u)}{f(v)} \right] \pmod{3}$ where $f(u) \geq f(v)$. If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G . $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$. Here we proved that C_n and some cycle related graphs like $[C_n; C_3]$, $n=3,6,9,\dots$, $(P_2 \cup mk_1) + N_2$, $S(C_n; S_2)$, joint sum of C_n and two cycles C_n having a common vertex is quotient-3 cordial.

Key words: cycle, joint sum, subdivision, quotient-3 cordial graph.

1. Introduction

Here the graphs considered are finite, simple, undirected and non trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [2] for more information. The cordial labeling concept was first introduced by Cahit [1]. The quotient-3 cordial labeling have been introduced by P. Sumathi, A. Mahalakshmi and A. Rathi found in [5-7]. They found some family of graphs are quotient-3 cordial. For notations and terminology we follow [8]. If G receives quotient-3 cordial labeling then G is called as quotient-3 cordial graph. The number of vertices having label i denotes $v_f(i)$ and the number of edges having label k denotes $e_f(k)$, $1 \leq i \leq 3$, $0 \leq k \leq 2$.

2. Preliminaries

Definition: 2.1 $[C_n; C_3]$ graph is obtained by attaching the cycle C_3 with every vertex of C_n .

Definition: 2.2

$(P_2 \cup mk_1) + N_2$ graph is obtained with the vertex set $V = \{z_1, z_2, x_1, x_2, \dots, x_m\} \cup \{y_1, y_2\}$ and the edge set $\{(y_1z_1), (y_1z_2), (y_2z_1), (y_2z_2), (z_1z_2)\} \cup \{(y_1x_1) \cup (y_2x_i) : 1 \leq i \leq m\}$.

Definition: 2.3

A graph $(C_n; S_2)$ is obtained by attaching the star S_2 with each vertex of a cycle C_n through an edge.

Definition: 2.4 From G , a graph $S(G)$ is obtained by subdividing every edge of G with a new vertex is called subdivision of G .

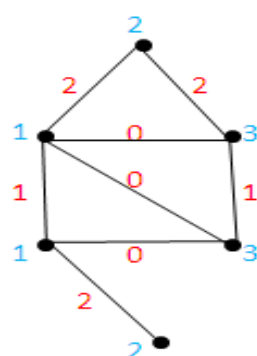
Definition: 2.5

A vertex of first copy of C_n is connected with a vertex of second copy of C_n through an edge is said to be joint sum of C_n .

3. Main Result

Definition: Let G be a graph of order p and size q . Let $f: V(G) \rightarrow Z_4 - \{0\}$ be a function. For each $E(G)$ define $f^*: E(G) \rightarrow Z_3$ by $f^*(uv) = \left[\frac{f(u)}{f(v)} \right] \pmod{3}$ where $f(u) \geq f(v)$. If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G . $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$.

Illustration: 1 A quotient-3 cordial graph



Theorem: 3.1 All cycles C_n are quotient-3 cordial for $n \geq 4$ ($n \neq 9, 15, 21, \dots$)

Proof: Let $V(G) = \{x_i : 1 \leq i \leq n\}$ and $E(G) = \{(x_n x_1), (x_i x_{i+1}) : 1 \leq i \leq n-1\}$.

Here $|V(G)| = |E(G)| = n$.

Let $f: V(G) \rightarrow Z_4 - \{0\}$

Case (i): when $n \equiv 0, 1, 4, 5 \pmod{6}$

For all i ,

$f(x_i) = 1, \quad i \equiv 1, 2 \pmod{6}$

$f(x_i) = 3, \quad i \equiv 0, 3 \pmod{6}$

$f(x_i) = 2, \quad i \equiv 4, 5 \pmod{6}$

Case (ii): when $n \equiv 2 \pmod{6}$

Labeling of $x_i, 1 \leq i \leq n-3$ are same as in case (i).

In this case, label the vertices x_{n-2}, x_{n-1}, x_n by 2, 1 and 3 respectively.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1, 4 \pmod{6}$	$\frac{n+2}{3}$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3} - 1$
$n \equiv 2, 5 \pmod{6}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3} - 1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{6}$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod{6}$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3}$	$\frac{n+2}{3} - 1$
$n \equiv 2 \pmod{6}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3} - 1$
$n \equiv 4 \pmod{6}$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3} - 1$	$\frac{n+2}{3}$
$n \equiv 5 \pmod{6}$	$\frac{n+1}{3} - 1$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

Table 1

Theorem: 3.2 The graph $[C_n; C_3]$ is quotient-3 cordial for $n = 3, 6, 9, \dots$

Proof: Let $V(G) = \{[x_i: 1 \leq i \leq n] \cup [v_{ij}: 1 \leq i \leq n, j = 1, 2]\}$

$E(G) = \{[(x_1x_n), (x_ix_{i+1}): 1 \leq i \leq n-1] \cup [(x_iv_{ij}): 1 \leq i \leq n, j = 1, 2] \cup [(v_{i1}v_{i2}): 1 \leq i \leq n]\}$

Let $|V(G)| = 3n, |E(G)| = 4n$.

Define $f: V(G) \rightarrow Z_4 - \{0\}$

Labeling of $x_i, 1 \leq i \leq n$ is given below.

$f(x_i) = 1, i \equiv 1, 2 \pmod{3}$

$f(x_i) = 3, i \equiv 0 \pmod{3}$

Labeling of $v_{ij}, 1 \leq i \leq n, 1 \leq j \leq 2$ is given below

$f(v_{ij}) = 1, i \equiv 1 \pmod{3}, j = 1$

$f(v_{ij}) = 3, i \equiv 1 \pmod{3}, j = 2$

$f(v_{ij}) = 3, i \equiv 0 \pmod{3}, j = 1$

$f(v_{ij}) = 2, i \equiv 2 \pmod{3}, j = 1, 2$

$f(v_{ij}) = 2, i \equiv 0 \pmod{3}, j = 2$

For all $n, v_f(1) = v_f(2) = v_f(3) = n$

$$e_f(0) = e_f(1) = e_f(2) = \frac{4n}{3}.$$

Theorem: 3.3 A graph $(P_2 \cup nK_1) + N_2$ is quotient-3 cordial ($n \neq 2, 5, 8, \dots$)

Proof: Let $V(G) = \{u_i, v_i, w_j: 1 \leq j \leq n, i = 1, 2\}$ and

$E(G) = \{[(u_1u_2), (v_1u_1), (v_1u_2), (v_2u_1), (v_2u_2)] \cup [(v_1w_j), (v_2w_j): 1 \leq j \leq n]\}$

Let $|V(G)| = 4 + n, |E(G)| = 5 + 2n$.

Let $f: V(G) \rightarrow Z_4 - \{0\}$

$f(u_1) = f(v_1) = 1$

$f(u_2) = f(v_2) = 3$.

Label the vertices $w_j, 1 \leq j \leq n$ as follows

$f(w_1) = 2$

When $n \equiv 0, 1 \pmod{3}$

For $2 \leq j \leq n$,

$f(w_j) = 1, j \equiv 2 \pmod{3}$

$f(w_j) = 2, j \equiv 0 \pmod{3}$

$f(w_j) = 3, j \equiv 1 \pmod{3}$

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{6}$	$\frac{n+3}{3} + 1$	$\frac{n+3}{3}$	$\frac{n+3}{3}$
$n \equiv 1 \pmod{6}$	$\frac{n+2}{3} + 1$	$\frac{n+2}{3}$	$\frac{n+2}{3} + 1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{6}$	$\frac{2n+3}{3} + 1$	$\frac{2n+3}{3}$	$\frac{2n+3}{3} + 1$
$n \equiv 1 \pmod{6}$	$\frac{2n+1}{3} + 1$	$\frac{2n+1}{3} + 1$	$\frac{2n+1}{3} + 1$

Table 2

Theorem: 3.4 $S(C_n; S_2)$ are quotient-3 cordial.

Proof: The cycle C_n has the vertices $u_1, u_2, u_3, \dots, u_n$. Let x_i be the vertices subdividing the edges $(u_i u_{i+1})$ for all $1 \leq i \leq n-1$ and the vertex x_n subdividing the edge $u_n u_1$.

Let $V(G) = \{[u_i, x_i: 1 \leq i \leq n] \cup [v_{ij}: 1 \leq i \leq n, j = 1, 2] \cup [w_{ij}: 1 \leq i \leq n, j = 1, 2]\}$ and
 $E(G) = \{[(u_n x_n), (x_n u_1), (u_i x_i), (x_i u_{i+1}): 1 \leq i \leq n-1] \cup [(u_{2i-1} v_{ij}): 1 \leq i \leq n, j = 1, 2] \cup [v_{ij} w_{ij}): 1 \leq i \leq n, j = 1, 2]\}$

Let $|V(G)| = |E(G)| = 6n$.

Define $f: V(G) \rightarrow Z_4 - \{0\}$ by

For all $i, 1 \leq i \leq n$ and $j = 1, 2$

$f(u_i) = f(x_i) = 3$

$f(v_{ij}) = 1$

$f(w_{ij}) = 2$

For all $n, v_f(1) = v_f(2) = v_f(3) = 2n$

$e_f(0) = e_f(1) = e_f(2) = 2n$.

Theorem: 3.5 Joint sum of C_n are quotient-3 cordial.

Proof: The first cycle C_n has the vertices $u_1, u_2, \dots, u_n$ and an another cycle C_n has the vertices $u_{n+1}, u_{n+2}, \dots, u_{2n}$.

Let $V(G) = \{[u_i: 1 \leq i \leq 2n]\}$.

$E(G) = \{[(u_1 u_n), (u_i u_{i+1}): 1 \leq i \leq n-1] \cup [(u_1 u_{n+1})] \cup [(u_i u_{i+1}), (u_{n+1} u_{2n}): n+1 \leq i \leq 2n-1]\}$

Let $|V(G)| = 2n$ and $|E(G)| = 1 + 2n$

Let $f: V(G) \rightarrow Z_4 - \{0\}$

Case (i): when $n \equiv 0, 2, 3, 5 \pmod{6}$

For $1 \leq i \leq 2n$

$f(u_i) = 1, i \equiv 1, 2 \pmod{6}$

$f(u_i) = 3, i \equiv 0, 3 \pmod{6}$

$f(u_i) = 2, i \equiv 4, 5 \pmod{6}$

Case (ii): when $n \equiv 1 \pmod{6}$

For $1 \leq i \leq n$

$f(u_i) = 2, i \equiv 1, 2 \pmod{6}$

$f(u_i) = 3, i \equiv 0, 3 \pmod{6}$

$f(u_i) = 1, i \equiv 4, 5 \pmod{6}$

For $n+1 \leq i \leq 2n$

$f(u_i) = 3, i \equiv 1, 4 \pmod{6}$

$f(u_i) = 1, i \equiv 2, 3 \pmod{6}$

$f(u_i) = 2, i \equiv 0, 5 \pmod{6}$

In this case after labeling all the vertices interchange the label of u_{n+3} and u_{n+4} .

Case (iii): when $n \equiv 4 \pmod{6}$

Labeling of the vertices u_i , $1 \leq i \leq 2n-1$ [$i \neq n$] are same as in Case (i)

Here $f(u_n) = 3$, $f(u_{2n}) = 2$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0,3 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod{6}$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
$n \equiv 2 \pmod{6}$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
$n \equiv 4 \pmod{6}$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$
$n \equiv 5 \pmod{6}$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3} + 1$	$\frac{2n}{3}$
$n \equiv 1,4 \pmod{6}$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
$n \equiv 2 \pmod{6}$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$	$(\frac{2n+2}{3})$
$n \equiv 3 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3} + 1$
$n \equiv 5 \pmod{6}$	$(\frac{2n+2}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$

Table 3

Theorem: 3.6 Two copies of cycle C_n having a common vertex is quotient-3 cordial.

Proof: Let $V(G) = \{[u_i: 1 \leq i \leq 2n-1]\}$ and

$E(G) = \{[(u_1u_n), (u_iu_{i+1}): 1 \leq i \leq n-1] \cup [(u_1u_{n+1}), (u_iu_{2n-1}), (u_iu_{i+1}): n+1 \leq i \leq 2n-2]\}$

Let $|V(G)| = 2n - 1$ and $|E(G)| = 2n$

Let $f: V(G) \rightarrow \mathbb{Z}_4 - \{0\}$

Case (i): When $n \equiv 0 \pmod{6}$

$f(u_{2n-1}) = 3$.

For $1 \leq i \leq 2n-2$

$f(u_i) = 1$, $i \equiv 1, 2 \pmod{6}$

$f(u_i) = 3$, $i \equiv 0, 3 \pmod{6}$

$f(u_i) = 2$, $i \equiv 4, 5 \pmod{6}$

Case (ii): when $n \equiv 1 \pmod{6}$

$f(u_{2n-3}) = 2$

$f(u_{2n-2}) = 1$

$f(u_{2n-1}) = 3$

For $1 \leq i \leq 2n-4$

$f(u_i) = 1$, $i \equiv 0, 1 \pmod{6}$

$f(u_i) = 3$, $i \equiv 2, 5 \pmod{6}$

$f(u_i) = 2$, $i \equiv 3, 4 \pmod{6}$

Case (iii): when $n \equiv 2 \pmod{6}$

$f(u_{2n-2}) = 2$.

Labeling of the vertices u_i , $1 \leq i \leq 2n-1$ ($i \neq 2n-2$) are same as in case (i).

Case (iv): when $n \equiv 3 \pmod{6}$

$$f(u_{2n-1}) = 2.$$

Labeling of the vertices u_i , $1 \leq i \leq 2n-2$ are same as in case (i).

Case (v): when $n \equiv 4 \pmod{6}$

Labeling of the vertices u_i , $1 \leq i \leq n-1$ are same as in case (i) and $f(u_n) = 3$.

Labeling of the vertices u_i , $n+1 \leq i \leq 2n-1$ is given below.

For $1 \leq i \leq 2n-1$

$$f(u_i) = 2, \quad i \equiv 1, 2 \pmod{6}$$

$$f(u_i) = 3, \quad i \equiv 0, 3 \pmod{6}$$

$$f(u_i) = 1, \quad i \equiv 4, 5 \pmod{6}$$

Case (vi): when $n \equiv 5 \pmod{6}$

$$f(u_{2n-3}) = 3, f(u_{2n-2}) = 1, f(u_{2n-1}) = 2.$$

Labeling of the vertices u_i , $1 \leq i \leq 2n-4$ are same as in case (i).

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3} - 1$	$\frac{2n}{3}$
$n \equiv 1 \pmod{6}$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 2, 5 \pmod{6}$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
$n \equiv 3 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3} - 1$
$n \equiv 4 \pmod{6}$	$(\frac{2n-2}{3})$	$(\frac{2n-2}{3})$	$(\frac{2n+1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0, 3 \pmod{6}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod{6}$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 2 \pmod{6}$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$
$n \equiv 4 \pmod{6}$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n-2}{3})$
$n \equiv 5 \pmod{6}$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n+2}{3})$

Table 4

4. Conclusion

In this paper, a quotient-3 cordial for some standard graphs has been found. The quotient-3 cordial labeling of some more graphs and graph families shall be explored in future.

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