ISSN(Online) : *2456-8910*
International Journal of Innovative Research in Applied Sciences and Engineering (IJIRASE)
Volume 1, Issue 3, DOI: 10.29027/IJIRASE.v1.i3.2017.80-85, September 2017

Quotient Labeling of Corona of Ladder Graphs

Dr.P. Sumathi[1], A. Rathi[2*] and A. Mahalakshmi[3]

[1]Department of Mathematics, C. Kandaswami Naidu College for Men, Anna Nagar, Chennai 600 102, India. Email : sumathipaul@yahoo.co.in

[2*]Department of Mathematics, Dr. M.G.R. Educational and Research Institute, University, Maduravoyal, Chennai 600 095, India., Email : rathi.math@drmgrdu.ac.in

[3]Department of Mathematics, Sri Muthukumaran Institute of Technology, Mangadu, Chennai 600122, India. Email :mahalakshmia.math@gmail.com

Abstract – Let G (V, E) be a finite, non-trivial, simple and undirected graph of order n and size m. For an one to one assignment f: V(G)→{1,2,…., n}, A Quotient labeling f* : $E(G)$→ {1, 2, ... , n} defined by $f^*(uv)=\left\lfloor \frac{f(u)}{f(v)} \right\rfloor$ where f(u) > f(v), then the edge labels need not be distinct. The q-labeling number $q_l(f^*)$ is the maximum value of f*(E(G)), and the Quotient Labeling Number $Q_L(G)$ is the minimum among$q_l(f^*)$. The bounds for the Quotient labeling number for corona ofsome ladder graphs likeclosed ladder,open ladder, slanting ladder, open triangular ladder, closed triangular ladder and diagonal ladder are found in this paper.

Key words: Quotient labeling number; open ladder graph; closed ladder; corona; slanting ladder; triangular ladder; diagonal ladder.

Introduction: The graph labeling problemswasinitially introduced by Alex Rosa in the year 1967. Various types of graph labeling problems have been defined around this and is not only due to its mathematical importance but also because of having the wide range of applications. Every year a survey comes with the updating of various graph labeling problems by J. A. Gallian [1]. Quotient labeling of graphs was first introduced by P. Sumathi and A. Rathi[2]. They found the quotient labeling number of various graphs and are found in [3-5]. In this paper we found the quotientlabeling number of corona of family of ladder graphs. For Graph notations and terminology we follow [6].

Preliminaries: The graphs that we considered in this paper are finite, simple, non-trivial and undirected graphs. We use the following definition that are relevant to this paper.

Definition: The graph G ʘ mK_1 is the graph obtained from the graph G by addingm number of pendent vertices to every vertex of G. When m =1, G ʘ mK_1 is known as the corona of G.
Definition: A Closed Ladder graph L_n, n≥2 is obtained from two copies of the paths of length n-1withvertex set V(G) = {u_i, v_i :1≤ i ≤ n } and edge set E(G) = $\{u_iu_{i+1,}v_iv_{i+1} : 1 \le i \le n-1\}$ ∪ $\{u_iv_i$: $1 \le i \le$ n}.**Definition:**An Open ladder graph O(L_n), n≥2 is a ladder graph with 2n vertices and is got from two paths of length n-1with V(G) = {u_i, v_i :1≤ i ≤ n } and E(G) = $\{u_iu_{i+1,}v_iv_{i+1} : 1 \le i \le n-1\}$ ∪ $\{u_iv_i$: $2 \le i \le$n-1}.

Definition:A Slanting ladder graph SL_n, n≥2 is a ladder graph with 2n verticesand is obtained from two paths of length n-1with V(G) = {u_i, v_i :1≤ i ≤ n } and E(G) = $\{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} : 1 \leq i \leq$n-1 }.
Definition: AClosed Triangular ladder TL_n, n≥2 is a ladder graphwith 2n vertices and is got from aClosed Ladder graph $L_n$with the additional edges u_iv_{i+1} for 1≤i ≤n-1.
Definition: An open Triangular ladder O(TL_n), n≥2 is a ladder graph with 2n vertices and is got from an open ladder O(L_n)with the additional edges u_iv_{i+1} for 1≤ i ≤ n-1.
Definition: Diagonal ladder graph DL_n, n ≥2is a ladder graph with 2n vertices and is got from a ladder graph L_n with the additional edges u_iv_{i+1} and $u_{i+1}v_i$ for 1≤ i ≤ n-1.

Theorem: 1The quotient labeling number of ($\boldsymbol{L_n}$⊙ K1)is 2.Proof: LetG =($\boldsymbol{L_n}$⊙ K_1)be the given graph with 4n vertices. The graph G is got by attaching one new vertex to every vertex of $\boldsymbol{L_n}$.
Let $v_1, v_2 ....,v_n$ be the vertices on one side of $\boldsymbol{L_n}$ and $u_1, u_2, ..., u_n$ be the vertices on the other side of$\boldsymbol{L_n}$.
Let x_1, x_2, …, x_n be the n new vertices attached with the corresponding n vertices $v_1, v_2, ..., v_n$ and y_1, y_2, …, y_n be the n new vertices attached with the corresponding n vertices $u_1, u_2, ..., u_n$ respectively.
Now the graph has 4n vertices with deg v_1 =deg v_n =degu_1=deg u_n= 3, deg v_i=deg u_i = 4 for 2≤ i ≤ n-1 and deg x_i = deg y_i =1 for 1≤ i ≤ n, Let the function f : V(G) →{1,2,….,4n} be defined byf(x_1)=1
f(x_i)= 4i-3 for 2≤ i ≤ n
f(v_i)= i +1 for i =1,2.
f(v_i)= 4(i -1)-2 for 3≤ i ≤ nf(u_1)= 4, f(u_n)= 4n-2
f(u_i)= 4i-1 for 2≤ i ≤ n-1
f(y_i)= 4(i+1) for 1≤ i ≤ n-2
f(y_{n-1})= 4n-1, f(y_n)= 4n.
For the above vertex labeling we get f*(E(G)) ={ 1, 2}
Therefore the maximum value of f*(E(G)) is equal to 2.
Then q_l(f*) =2. But in G, δ(G) =1 and Δ(G) =4. Therefore q_l(f*) can take the value 2 or 3 or 4 or 5. Here q_l(f*) =2 and is minimum. Hence Q_L(G)= 2.

Theorem: 2 The quotient labeling number of the corona of an open ladder graphOL_nʘ$K_1$is 2.Proof: Let G = OL_nʘ K_1 be the corona of an open ladder graph.
Let $v_1, v_2 ....v_n$ be the vertices on one side of the ladder and $u_1, u_2, ..., u_n$ be the vertices on the other side of OL_n.The graph G is got by attaching one new vertex to every vertex of the open ladder graph OL_n.Let x_1, x_2, …, x_n be the n new vertices attached with the corresponding n vertices $v_1, v_2, ..., v_n$and let y_1, y_2, …, y_n be the n new vertices attached with the corresponding n vertices $u_1, u_2, ..., u_n$ respectively.

Now the graph G has 4n vertices with deg x_i = deg y_i =1 for $1\le i \le n$,
deg u_1 =deg u_n =deg v_1 =deg v_n =2 and deg u_i = deg v_i = 4 for $2\le i \le n-1$
Let the function f: V (G) → {1, 2, …., 4n} be defined by
$f(x_1)$ =1, $f(x_2)$ =4, $f(x_i)$ =5(i -2)+2 for i = $3\le i \le 5$
$f(x_i)$ =4i-3 for $6\le i \le n$
$f(v_i)$ =i+1 for i = 1,2.
$f(v_3)$ =5, $f(v_i)$ =5(i -3)+3 for i = $3\le i \le 5$.
$f(v_i)$ =4(i -2)+2 for $7\le i \le n$.
$f(u_1)$ =10, $f(u_2)$ =6, $f(u_i)$ =5(i -1)-1 for $3\le i \le 5$. $f(u_i)$ =4i-1 for $6\le i \le n-1$, $f(u_n)$ = 4n-2.
$f(y_1)$ =16, $f(y_2)$ =11,$f(y_3)$ =15,$f(y_i)$ =4(i +1) for$4\le i \le n-2$,$f(y_{n-1})$=4n-1, $f(y_n)$=4n. For the above vertex labeling we get f*(E(G)) ={ 1, 2} Therefore the maximum value of f*(E(G)) is equal to 2.
Then q_l(f*) =2. But in G, δ(G) =1 and Δ(G) =4. Therefore q_l(f*) can take the value 2 or 3 or 4 or 5. Here q_l(f*) =2 and is minimum. Hence Q_L(G)=2.

Theorem: 3 The quotient labeling number of corona of a slanting ladder graph SL_nʘ K_1 is 2.

Proof: let G=SL_nʘ K_1 be the given graph.
Let $v_1, v_2 \dots v_n$ be the vertices on one side of the ladder and $u_1, u_2, \dots, u_n$ be the vertices on the other side of the slanting ladder and each v_i is adjacent with u_{i+1} for $1\le i \le n-1$..The graph G is got by attaching one new vertex to every vertex of the slanting ladder graph SL_n.
Let $x_1, x_2, \dots, x_n$ be the n new vertices attached with the corresponding n vertices $v_1, v_2, \dots, v_n$ and let $y_1, y_2, \dots, y_n$ be the n new vertices attached with the corresponding n vertices $u_1, u_2, \dots, u_n$ respectively.
Now the graph G has 4n vertices with deg x_i = deg y_i =1 for $1\le i \le n$,
deg v_n =deg u_1 =2 , deg v_1 =deg u_n =3 and deg u_i = deg v_i = 4 for $2\le i \le n-1$
Let the function f: V (G) → {1, 2, …., 4n} bedefined by
$f(x_1)$ =1, $f(x_2)$ = 8,
$f(x_i)$ =4i+1 for $3\le i \le n-1$
$f(x_n)$ =4n
$f(v_i)$=2i for i =1,2.
$f(v_3)$= 9 $f(v_i)$=4i-2 for $4\le i \le n$
$f(u_1)$ =5$f(u_2)$ =3$f(u_3)$ =7
$f(u_i)$=4(i -1) for $4\le i \le n$.
$f(y_1)$=10, $f(y_2)$=6
$f(y_i)$=4i-1 for $3\le i \le n$
For the above vertex labeling f*(E(G)) = {1, 2}.
Therefore the maximum value of f*(E(G)) is equal to 2.
Then q_l(f*) =2. But in G, δ(G) =1 and Δ(G) =4. Therefore q_l(f*) can take the values 2 or 3 or 4 or 5. Here q_l(f*) =2 and is minimum. Hence Q_L(G)=2.

Theorem: 4 The quotient labeling number of the corona of any closed triangular ladder $TL_n \odot$ K1, n≥2 is 2.

Proof: let G =$TL_n \odot K_1$ be the corona of a closed triangular ladder graph.

Let $v_1, v_2 \dots v_n$ be the vertices on one side of the closed triangular ladder TL_n and $u_1, u_2, \dots, u_n$ be the vertices on the other side of TL_n.The graph G is got by attaching one new vertex to every vertex of the ladder graph TL_n.

Let $x_1, x_2, \dots, x_n$ be the n new vertices attached with the corresponding n vertices $v_1, v_2, \dots, v_n$ and let $y_1, y_2, \dots, y_n$ be the n new vertices attached with the corresponding n vertices $u_1, u_2, \dots, u_n$ respectively.

Now the graph G has 4n vertices with deg x_i = deg y_i =1 for $1\le i \le n$,

deg v_1 =deg u_n =4, deg u_1 =deg v_n =3 and deg u_i = deg v_i = 5 for $2\le i \le n-1$. Let the function f: V (G) → {1, 2, …., 4n}be definedby

$f(x_1) = 1$, $f(x_i) =4i-1$ for $2\le i \le n$.$f(v_1)=2$, $f(v_i)=4(i-1)$ for$2\le i \le n$.

$f(u_1) =3$, $f(u_i)=4i-3$ for$2\le i \le n$

$f(y_i)=4i+2$ for $2\le i \le n-1$ and $f(y_n)=4n$.For the above vertex labeling we get f*(E(G)) = {1, 2}. Therefore the maximum value of f*(E(G)) is equal to 2.

Then $q_l(f^*)$ =2. But in G, δ(G) =1 and Δ(G) =5. Therefore $q_l(f^*)$ can take the values 2 or 3 or 4 or 5 or 6. Here $q_l(f^*)$ =2 and is minimum. Hence $Q_L(G)$=2.

Theorem: 5 The quotient labeling number of the corona of any open triangular ladder $OTL_n \odot$ K1, n≥2 is 2.

Proof: let G=$OTL_n \odot K_1$ be the given graph. Let $v_1, v_2 \dots v_n$ be the vertices on one side of the open triangular ladder OTL_n and $u_1, u_2, \dots, u_n$ be the vertices on the other side of OTL_n.The graph G is got by attaching one new vertex to every vertex of the ladder graph OTL_n.

Let $x_1, x_2, \dots, x_n$ be the n new vertices attached with the corresponding n vertices $v_1, v_2, \dots, v_n$ and let $y_1, y_2, \dots, y_n$ be the n new vertices attached with the corresponding n vertices $u_1, u_2, \dots, u_n$ respectively.

Now the graph G has 4n vertices with deg x_i = deg y_i =1 for $1\le i \le n$,

deg v_1 =deg u_n =3 , deg u_1 =deg v_n =2 and deg u_i = deg v_i = 5 for $2\le i \le n-1$

Let the functionf: V (G) → {1, 2, …., 4n}be definedby $f(x_1)$ =1, $f(x_2) = 5(i -1)$ for i = 2, 3.

$f(x_i) =4i-1$ for $4\le i \le n$

$f(v_i)=i+1$ for i =1,2.

$f(v_i)=5(i -2)+1$ fori = 3,4.

$f(v_i)=4(i -1)$ for$5\le i \le n$

$f(u_1) =8$,$f(u_2) =4$,$f(u_3) =7$ $f(u_i)=5(i -2)+2$ fori = 3,4.

$f(u_i)=4(i -1)+1$ for $6\le i\le n$.

$f(y_1)=14$, $f(y_2)=9$, $f(y_3)=13$,

$f(y_i)=4i+2$ for $4\leq i \leq n-1$ and $f(y_n)=4n$.For the above vertex labeling $f^*(E(G)) = \{1, 2\}$.
Therefore the maximum value of $f^*(E(G))$ is equal to 2.
Then $q_l(f^*) =2$. But in G, $\delta(G) =1$ and $\Delta(G) =5$. Therefore $q_l(f^*)$ can take the values 2 or 3 or 4 or 5 or 6. Here $q_l(f^*) =2$ and is minimum. Hence $Q_L(G)= 2$.

Theorem: 6 The quotient labeling number of corona of a diagonal ladder graph $DL_n \odot K_1$ is 2.Proof: Let $G = DL_n \odot K_1$be the corona of a diagonal ladder graph.
Let $v_1, v_2 \ldots v_n$ be the vertices on one side of the open triangular ladder DL_n and $u_1, u_2, \ldots, u_n$ be the vertices on the other side of DL_n.The graph G is got by attaching one new vertex to every vertex of the ladder graph DL_n.
Let $x_1, x_2, \ldots, x_n$ be the n new vertices attached with the corresponding n vertices $v_1, v_2, \ldots, v_n$ and let $y_1, y_2, \ldots, y_n$ be the n new vertices attached with the corresponding n vertices $u_1, u_2, \ldots, u_n$ respectively.
Now the graph G has 4n vertices with $\deg x_i = \deg y_i =1$ for $1\leq i \leq n$,
$\deg v_1 =\deg u_n = \deg u_1 =\deg v_n =4$ and $\deg u_i = \deg v_i = 6$ for $2 \leq i \leq n -1$.Let the function $f: V(G) \rightarrow \{1, 2, \ldots, 4n\}$ be defined by$f(x_1) =1$, $f(x_2) = 6$, $f(x_i) =4i -1$ for $3\leq i \leq n$
$f(v_i)=i +1$ for $i =1,2$.
$f(v_3) = 7$, $f(v_i)=4(i -1)$ for$4\leq i \leq n$
$f(u_1) = 5$,$f(u_2) =4$,$f(u_3) =8$$f(u_i)=4i -3$ for $4\leq i \leq n$.
$f(y_1)=10$, $f(y_2)=9$,
$f(y_i)=4i+2$ for $3\leq i \leq n-1$ and $f(y_n)=4n$.For the above vertex labeling $f^*(E(G)) = \{1, 2\}$.
Therefore the maximum value of $f^*(E(G))$ is equal to 2.
Then $q_l(f^*) =2$. But in G, $\delta(G) =1$ and $\Delta(G) =6$. Therefore $q_l(f^*)$ can take the values 2 or 3 or 4 or 5 or 6 or 7. Here $q_l(f^*) =2$ and is minimum. Hence $Q_L(G)=2$.

Conclusion: Quotient labeling number has been calculated for some family of graphs and the future work is to find quotient labeling number for other new classes of graphs.

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