Mod(k) Vertex Magic Labeling in Generalized 2-complement of some Graphs- Paper II

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ABSTRACT

A (p,q) graph G with the p vertices and q edges is Mod(k) vertex magic for any integer k≥2,l∈Zk and there exists a injective map f from V(G) to {\(\frac{k}{2}\), \(\frac{k}{2}+l\), \(\frac{k}{2}+l+1\),... \(\frac{k}{2}+(p-1)\)} such that for any edge e, and the sum of the labels of vertices adjacent with e are all equal to the same constant modulo k. In this paper, we prove that Generalized 2-complement some graphs namely S(K₁,n), Spl (Cₙ) are Mod(k) vertex magic graphs.

Keywords: Graph labeling, Mod(k) vertex magic labeling, star, Subdivision , Splitting graph.

AMS Subject Classification: 05C78

1.INTRODUCTION

In this paper, we manage just, connected and non-trivial graph G=(V(G),E(G)) among vertex set V(G) and edge set E(G).

Let G=(V,E) to be a graph and P=(W₁,W₂,W₃...Wₖ) be partition of V of order k>1. The k-complement GkP of G (concerning P) is characterized as takes after: For all Wi and Wj in P, i≠j expel the edges amongst Wi and Wj in G and join the edges amongst Wi and Wj which are not in G. The graph accordingly acquired is known as the k-complement of G regarding P [2].

A labeling of a graph G is a mapping that takes an set of graph components for the most part vertices and edges into an set of numbers, generally integers. Numerous sorts of labeling have been examined and a splendid overview of graph labeling is built up [4].

The idea of graph labeling has fulfilled a considerable measure of ubiquity in the area of graph theory. This graph labeling are extremely valuable in Mathematical models for an extensive variety of uses being X-ray, Crystallography , Coding theory, Cryptography, Communication networks design, Radar , Space science, Circuit design and, Database Administration.

In 1970, Kotzig and Rosa defined a magic labeling of a graph G(V,E) as abijection, f:V∪E→\{1,2,3... p+q\}such that for all edges uw, f(u)+f(w)+f(uw) are the equal [1].

Lee, Su, Wang in 2010 defined (p,q) graph G is called Mod(k) edge magic (in short Mod(k)-EM) if here was an edge labeling l:E(G)→\{1,2,3...q\}such that for any vertex u, sum of the
labels of their edges incident with the $u$ are all equal to the same constant modulo $k$. (i.e) $f'(u)=c$ for some fixed $c$ in $Z_k$ [8].

In 2015, Lau, Alikhani, Lee, Kocay characterized a $(p,q)$ graph to be $k$-edge magic if for any integer $k \geq 0$, define a one-to-one map guided from $E(G)$ to $\{k, k+1, \ldots, k+q-1\}$ and characterized the vertex total for a vertex $v$ as the total of the labels of the edges incident to $v$. In the event that such an edge labeling makes a vertex labeling in which every vertex has a constant vertex total $(\text{mod } p)$ [2].

In 2016, P.Sumathi and B.Fathima [5] characterized Mod($k$) Vertex magic labeling. A $(p,q)$ graph $G=(V,E)$ is said to be mod$(k)$ vertex magic if for any integer $k \geq 2, e \in Z_k$ and there exists an one-to-one map $f: V(G) \rightarrow \{\left\lfloor \frac{k}{2} \right\rfloor, \left\lfloor \frac{k}{2} \right\rfloor + l, \left\lfloor \frac{k}{2} \right\rfloor + l + 1, \left\lfloor \frac{k}{2} \right\rfloor + k, \left\lfloor \frac{k}{2} \right\rfloor + k + l, \left\lfloor \frac{k}{2} \right\rfloor + k + l + 1, \ldots \left\lfloor \frac{k}{2} \right\rfloor + k(p-1)\}$ to such an extent that the induced mapping $f^*: E(G) \rightarrow Z_k$ characterized by $f^*(uv)=(f(u)+f(v))(\text{mod } k)= l$ is a constant mapping. The function $f$ is known as a mod$(k)$ vertex magic labeling (in short Mod$(k)$ VML) of $G$.

In this paper, we contemplate mod$(k)$ vertex magic labeling of 2-complement of a few graphs to be specific $S(K_{1,n})$, Spl $(C_n)$.

2. PRELIMINARIES

In this section, we give the essential definitions and notations identified with this paper.

Definition 2.1. From the graph $G$, a new graph were obtained by subdividing any edge $G$ with a new vertex is called Subdivision of the $G$ and it were denoted by $S(G)$ [4].

Definition 2.2. For a graph $G$, the Splitting graph of $G$ ($\text{Spl}(G)$) were attained from the $G$ joining of any vertex $u$ of $G$ is the new vertex of $u'$ is adjacent to every vertex and is adjacent to $u$ [4].

3. MAIN RESULTS

In this section, we given the existence of Mod($k$) vertex magic labeling of 2-complement of some graphs.

Theorem: 3.1. Let $G$ be a Subdivision of a star $(S(K_{1,n}))$ with $(2n+1, n \geq 1)$ vertices say $\{u, u_1, u_2, u_3, \ldots, u_n, v_1, v_2, v_3, \ldots, v_n\}$. If $W_1=\{u\}$ and $W_2=\{u, v_1 : 1 \leq i \leq n\}$ be the partition of $G_2^p$ ($S(K_{1,n})$) then 2-complement($G_2^p$) of $S(K_{1,n})$ admits Mod($k$) vertex magic labeling.

Proof: Let $K_{1,n}$ be a star with $\{u\} \cup \{u_i, 1 \leq i \leq n\}$ be the vertices and $\{uu_i, 1 \leq i \leq n\}$ be the edges.

Let $G=S(K_{1,n})$ be the Subdivision of $K_{1,n}$ is attained by subdividing any edge of $K_{1,n}$ with a new vertex $\{v_1 : 1 \leq i \leq n\}$ where $V(G)=\{u\} \cup \{u_i, 1 \leq i \leq n\} \cup \{v_1, 1 \leq i \leq n\}$ and $E(G)=\{uu_i, 1 \leq i \leq n\} \cup \{u_i v_1, 1 \leq i \leq n\}$. It has $2n+1$ vertices and $2n$ edges. Let
$G_1=(V(G_1), E(G_1))$ be the 2-complement $(G_2^p)$ of $S(K_{1,n})$ has two partitions $W_1=\{u\}$ and $W_2=\{u_i, v_i, 1 \leq i \leq n\}$ where $V(G_1)=\{u\} \cup \{u_i, v_i, 1 \leq i \leq n\}$ and $E(G_1)=\{uv, 1 \leq i \leq n\}$.

**Case(i):** When $k$ is odd.

Define $f: V(G_1) \rightarrow \left\{ \frac{k}{2}, \frac{k}{2} + l, \frac{k}{2} + k, \frac{k}{2} + k + l, \ldots, \frac{k}{2} + k(2n) \right\}$ by

$$f(w) = \begin{cases} 
\frac{k}{2}, & \text{if } w = u, \text{ for } 0 \leq l \leq k - 1, \\
\frac{k}{2} + k(2i), & \text{if } w = u_i, \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\
\frac{k}{2} + k(i), & \text{if } w = u_i, \text{ for } l = k - 1, 1 \leq i \leq n, \\
\frac{k}{2} + k(2i - 2) + l, & \text{if } w = v_i, \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\
\frac{k}{2} + k(n + i), & \text{if } w = v_i, \text{ for } l = k - 1, 1 \leq i \leq n.
\end{cases}$$

Clearly the mapping $f$ is an injective and we get.

$$f(u)+f(v)=\begin{cases} 
k(i) + l, & \text{if } u = v, v = v_i, \text{ for } 0 \leq l \leq k - 2, 1 \leq i \leq n, \\
k(n + i) + k - 1, & \text{if } u = v, v = v_i, \text{ for } l = k - 1, 1 \leq i \leq n.
\end{cases}$$

By the definition of Mod($k$) vertex magic labeling, the induced mapping $f^*$ is a constant mapping.

Thus $f$ is a Mod($k$) vertex magic labeling.

2-complement $(G_2^p)$ of $S(K_{1,n})$ is Mod($k$)vertex magic graph if $k$ is odd.

**Case(ii):** When $k$ is even.

Define $f: V(G_1) \rightarrow \left\{ \frac{k}{2}, \frac{k}{2} + l, \frac{k}{2} + k, \frac{k}{2} + k + l, \ldots, \frac{k}{2} + k(2n) \right\}$ by

$$f(w) = \begin{cases} 
\frac{k}{2}, & \text{if } w = u, \text{ for } 0 \leq l \leq k - 1, \\
\frac{k}{2} + k(i), & \text{if } w = u_i, \text{ for } l = 0, 1 \leq i \leq n, \\
\frac{k}{2} + k(2i), & \text{if } w = u_i, \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n, \\
\frac{k}{2} + k(n + i), & \text{if } w = v_i, \text{ for } l = 0, 1 \leq i \leq n \\
\frac{k}{2} + k(2i - 2) + l, & \text{if } w = v_i, \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n.
\end{cases}$$

Obviously the mapping $f$ is an injective and we get,

$$f(u)+f(v)=\begin{cases} 
k(n + i + 1), & \text{if } u = v, v = v_i, \text{ for } l = 0, 1 \leq i \leq n \\
k(i) + l, & \text{if } u = v, v = v_i, \text{ for } 1 \leq l \leq k - 1, 1 \leq i \leq n.
\end{cases}$$
Since the definition of $\text{mod}(k)$ vertex magic labeling, the induced mapping $f^*$ is a constant mapping.

Under the mapping $f$, there exists $\text{Mod}(k)$ vertex magic labeling for $(G_2^P)$ of $S(K_{1,n})$. Thus $2$-complement $(G_2^P)$ of $S(K_{1,n})$ is $\text{Mod}(k)$ vertex magic graph if $k$ is even.

Hence $2$-complement $(G_2^P)$ of $S(K_{1,n})$ admits $\text{Mod}(k)$ vertex magic labeling.

**Illustration: 1.** The following figures show the $S(K_{1,3})$ and $2$-complement of $S(K_{1,3})$ is a $\text{Mod}(5)$ vertex magic for $l=2$.

![Figure 1](image.png)

**Theorem: 3.2**

Let $G$ be a $\text{Spl}(C_n)$ graph with $\{u_1, u_2, u_3, \ldots, u_n\}$ and $\{v_1, v_2, v_3, \ldots, v_n\}$ is the vertices and $n$ even for all $n \geq 6$. If $W_1 = \{u_i, v_i: i \text{ is odd}\}$ and $W_2 = \{u_i, v_i: i \text{ is even}\}$ be the partition of $G_2^P(\text{Spl}(C_n))$ then $2$-complement $G_2^P$ of $\text{Spl}(C_n)$ admits $\text{Mod}(k)$ vertex magic labeling.

**Proof:** Let $C_n$ be the cycle of length $n$ and $n$ is even for all $n \geq 6$. Let $\{u_i: 1 \leq i \leq n\}$ be the vertices and $\{u_i u_{i+1}: 1 \leq i \leq n\} \cup \{u_n u_1\}$ be the edges of $C_n$.

Let $G = \text{Spl}(C_n) = (V(G), E(G))$ is obtained by adding each vertex $\{u_i: 1 \leq i \leq n\}$ of $C_n$ a new vertex $\{v_i: 1 \leq i \leq n\}$ adjacent to each vertex that is adjacent to the $\{u_i: 1 \leq i \leq n\}$ where $V(G) = \{u_i: 1 \leq i \leq n\} \cup \{v_i: 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}: 1 \leq i \leq n - 1\} \cup \{v_i u_{i+1}: 1 \leq i \leq n - 1\} \cup \{u_n u_1, v_n u_1, v_1 u_n\}$. It has $2n$ vertices and $3n$ edges.
Let $G_1=(V(G_1), E(G_1))$ be the 2-complement $(G_2^P)$ of Spl $(C_n)$ has two partitions $W_1=\{u_1,u_3,u_5,...,u_{2n-1},v_1,v_3,v_5,...,v_n\} \text{ and } W_2=\{u_2,u_4,u_6,...,u_n,v_2,v_4,v_6,...,v_n\}$ where

$V(G_1)= \{u_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n\} \text{ and } E(G_1)= E_1(G_1) \cup E_2(G_1) \cup E_3(G_1) \cup E_4(G_1) \cup E_5(G_1) \cup E_6(G_1) \cup E_7(G_1) \cup E_8(G_1) \cup E_9(G_1) \cup E_{10}(G_1)$ where

$E_1(G_1)= \{u_1 u_{2i+2}; 1 \leq i \leq \frac{n-4}{2}\}$, $E_2(G_1)= \{u_i u_{i-(2j+1)}; i=5,7,9,...,n-1, 1 \leq j \leq \frac{i-3}{2}\}$,

$E_3(G_1)= \{u_i u_{i+2j+1}; i=3,5,7,...,n-3, 1 \leq j \leq \frac{n-(i+1)}{2}\}$, $E_4(G_1)= \{u_1 v_{2i+2}; 1 \leq i \leq \frac{n-4}{2}\}$,

$E_5(G_1)= \{u_i v_{i-(2j+1)}; i=5,7,9,...,n-1, 1 \leq j \leq \frac{n-(i+1)}{2}\}$,

$E_6(G_1)= \{u_i v_{i+2j+1}; 1 \leq i \leq \frac{n-4}{2}\}$, $E_8(G_1)= \{v_i u_{i-(2j+1)}; i=5,7,9,...,n-1, 1 \leq j \leq \frac{i-3}{2}\}$,

$E_9(G_1)= \{v_i u_{i+2j+1}; i=3,5,7,...,n-3, 1 \leq j \leq \frac{n-(i+1)}{2}\}$, $E_{10}(G_1)= \{v_i v_{2i}; i \text{ is odd}\}$.

**Case(i):** When $k$ is odd.

Define $f: V(G_1) \rightarrow \{\frac{k}{2}, \frac{k}{2}+l+1, \frac{k}{2}+k, \frac{k}{2}+k+1, \frac{k}{2}+k+2, \frac{k}{2}+k+3, \frac{k}{2}+k+4, ..., \frac{k}{2}+k(2n-1)\}$ by

$$f(w)=\begin{cases} 
\frac{k}{2} + \frac{k}{2} (i - 1), \text{if } w = u_i \text{ for } 0 \leq l \leq k - 2, i \text{ is odd}, \\
\frac{k}{2} + \frac{k}{2} (i - 2) + l + 1, \text{if } w = u_i \text{ for } 0 \leq l \leq k - 2, i \text{ is even}, \\
\frac{k}{2} + k (i - 1), \text{if } w = u_i \text{ for } l = k - 1, 1 \leq i \leq n, \\
\frac{k}{2} + k (n + i - 1), \text{if } w = v_i \text{ for } 0 \leq l \leq k - 2, i \text{ is odd}, \\
\frac{k}{2} + k (n + i - 2) + l + 1, \text{if } w = v_i \text{ for } 0 \leq l \leq k - 2, i \text{ is even}, \\
\frac{k}{2} + k (n + i - 1), \text{if } w = v_i \text{ for } l = k - 1, 1 \leq i \leq n.
\end{cases}$$

Hence $f$ is an one - to - one map and we get,

$$f(u)+f(v)=\begin{cases} 
k(i + 2) + l \text{ if } u = u_1, v = u_{2i+2} \text{ for } 0 \leq l \leq k - 2, 1 \leq j \leq \frac{n-4}{2}, \\
k(2i + 1) + k - 1 \text{ if } u = u_1, v = u_{2i+2} \text{ for } l = k - 1, 1 \leq j \leq \frac{n-4}{2}, \\
k(i - j - 1) + l \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } 0 \leq l \leq k - 2, i = 5,7,9 ... n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(2i - 2j - 3) + k - 1 \text{ if } u = u_i, v = u_{i-(2j+1)} \text{ for } l = k - 1, i = 5,7,9 ... n-3, 1 \leq j \leq \frac{i-3}{2}, \\
k(i + j + 2) + l \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } 0 \leq l \leq k - 2, i = 3,5,7, ... n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(2i + 2j - 1) + k - 1 \text{ if } u = u_i, v = u_{i+2j+1} \text{ for } l = k - 1, i = 3,5,7, ... n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(2i + 2j) + l \text{ if } u = u_1, v = v_{2i+2} \text{ for } 0 \leq l \leq k - 2, 1 \leq j \leq \frac{n-4}{2}, \\
k(n + 2i + 1) + k - 1 \text{ if } u = u_1, v = v_{2i+2} \text{ for } l = k - 1, 1 \leq j \leq \frac{n-4}{2}.
\end{cases}$$
\[
f(u)+f(v) = \begin{cases} 
\frac{k}{2} (n + 2i - 2j - 2) + l, & \text{if } u = u_i, v = v_{i-(2j+1)} \text{ for } 0 \leq l \leq k-2, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i - 2j - 3) + k - 1, & \text{if } u = u_i, v = v_{i-(2j)+1} \text{ for } l = k - 1, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i + 2j) + l, & \text{if } u = u_i, v = v_{i+2j+1} \text{ for } 0 \leq l \leq k - 2, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2j - 1) + k - 1, & \text{if } u = u_i, v = v_{i+2j+1} \text{ for } l = k - 1, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
\frac{k}{2} (n + 2i + 2) + l & \text{if } u = v_1, v = u_{2i+2} \text{ for } 0 \leq l \leq k - 2, 1 \leq j \leq \frac{n-4}{2}, \\
k(n + 2i + 1) + k - 1, & \text{if } u = v_1, v = u_{2i+2} \text{ for } l = k - 1, 1 \leq j \leq \frac{n-4}{2}, \\
\frac{k}{2} (n + 2i - 2j - 2) + l & \text{if } u = v_i, v = u_{i-(2j+1)} \text{ for } 0 \leq l \leq k - 2, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i - 2j - 3) + k - 1, & \text{if } u = v_i, v = u_{i-(2j)+1} \text{ for } l = k - 1, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i + 2j) + l, & \text{if } u = v_i, v = u_{i+2j+1} \text{ for } 0 \leq l \leq k - 2, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2j + 1) + k - 1, & \text{if } u = v_i, v = u_{i+2j+1} \text{ for } l = k - 1, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
\frac{k}{2} (2n + 3i - 1), & \text{if } u = v_i, v = v_{2i} \text{ for } 0 \leq l \leq k - 2, i \text{ is odd,} \\
k(2n + 3i - 1), & \text{if } u = v_i, v = v_{2i} \text{ for } l = k - 1, i \text{ is odd.} 
\end{cases}
\]

Since the definition of Mod(k) vertex magic labeling, the induced mapping f* is a constant mapping.

Under the mapping f, there exists mod(k) vertex magic labeling for 2-complement of Spl (C_n).

Thus G^p_2 of Spl (C_n) is Mod(k) vertex magic graph if k is odd.

**Case(ii):** When k is even.

Define f: V(G)→\{[\frac{k}{2}], [\frac{k}{2}] + l, [\frac{k}{2}] +k, [\frac{k}{2}] +k+ l, \ldots [\frac{k}{2}] +k(2n-1)\} by

\[
f(w) = \begin{cases} 
\frac{k}{2} + k \ (i-1), & \text{if } w = u_l \text{ for } l = 0,1 \leq i \leq n, \\
\frac{k}{2} + \frac{k}{2} \ (i-1), & \text{if } w = u_l \text{ for } 1 \leq l \leq k-1, i \text{ is odd,} \\
\frac{k}{2} + \frac{k}{2} \ (i-2) + l, & \text{if } w = u_l \text{ for } 1 \leq l \leq k-1, i \text{ is even,} \\
\frac{k}{2} + k \ (n + i - 1), & \text{if } w = v_l \text{ for } l = 0,1 \leq i \leq n, \\
\frac{k}{2} + \frac{k}{2} \ (n + i - 1), & \text{if } w = v_l \text{ for } 1 \leq l \leq k-1, i \text{ is odd,} \\
\frac{k}{2} + \frac{k}{2} \ (n + i - 2) + l, & \text{if } w = v_l \text{ for } 1 \leq l \leq k-1, i \text{ is even.} 
\end{cases}
\]
Clearly $f$ is an injective mapping and we get,
\[
f(u)+f(v) = \\
k(2i + 2), \text{if } u = u_i, v = u_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}, \\
k(i + 2) + l \text{ if } u = u_i, v = u_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\
k(2i - 2j - 2), \text{if } u = u_i, v = u_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(i - j - 1) + l, \text{if } u = u_i, v = u_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(2i + 2j), \text{if } u = u_i, v = u_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(i + j + 2) + l, \text{if } u = u_i, v = u_{i+j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2) \text{ if } u = u_i, v = v_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}. \\
k \frac{1}{2} (n + 2i + 2) + l \text{ if } u = u_i, v = v_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\
k(n + 2i - 2j - 2), \text{if } u = u_i, v = v_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k \frac{1}{2} (n + 2i - 2j - 2) + l, \text{if } u = u_i, v = v_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i + 2j), \text{if } u = u_i, v = u_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2j) + l, \text{if } u = u_i, v = v_{i+2j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2), \text{if } u = v_i, v = u_{2i+2} \text{ for } l = 0, 1 \leq j \leq \frac{n-4}{2}, \\
k \frac{1}{2} (n + 2i + 2) + l \text{ if } u = v_i, v = u_{2i+2} \text{ for } 1 \leq l \leq k-1, 1 \leq j \leq \frac{n-4}{2}, \\
k(n + 2i - 2j - 2), \text{if } u = v_i, v = u_{i-(2j+1)} \text{ for } l = 0, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k \frac{1}{2} (n + 2i - 2j - 2) + l, \text{if } u = v_i, v = u_{i-(2j+1)} \text{ for } 1 \leq l \leq k-1, i = 5,7,9 \ldots n-1, 1 \leq j \leq \frac{i-3}{2}, \\
k(n + 2i + 2j), \text{if } u = v_i, v = v_{i+2j+1} \text{ for } l = 0, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(n + 2i + 2j) + l, \text{if } u = v_i, v = v_{i+2j+1} \text{ for } 1 \leq l \leq k-1, i = 3,5,7, \ldots n-3, 1 \leq j \leq \frac{n-(i+1)}{2}, \\
k(2n + 3i), \text{if } u = v_i, v = v_{2i} \text{ for } l = 0, i \text{ is odd,} \\
k \frac{1}{2} (2n + 3i - 1), \text{if } u = v_i, v = v_{2i} \text{ for } 1 \leq l \leq k-1, i \text{ is odd.}
\]

By the definition of Mod($k$) vertex magic labeling, the induced mapping $f^*$ is a constant mapping.

Thus $f$ is a mod($k$) vertex magic labeling.

$G_2^P$ of Spl ($C_n$) is Mod($k$) vertex magic graph if $k$ is even.

Hence 2-complement $G_2^P$ of Spl ($C_n$) admits Mod($k$) vertex magic labeling for all $n \geq 6$. 

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Illustration: 2. The following figures show $\text{Spl}(C_8)$ and its 2-complement is a Mod(2) vertex magic graph for $l=1$.

<table>
<thead>
<tr>
<th>Figure 2</th>
<th>$G_2^p(\text{Spl}(C_6))$</th>
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4. CONCLUSION

In this paper we have discussed that Generalized 2-complement of some graphs are Mod(k) vertex magic graphs. Analogues work can be carried by us for other families also.

REFERENCES


