ISSN(Online) : *2456-8910*
International Journal of Innovative Research in Applied Sciences and Engineering (IJIRASE)
Volume 2, Issue 11, DOI: 10.29027/IJIRASE.v2.i11.2019, 372-375 , May 2019

Applications of Meshfree Particle Method in Structural Engineering

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***Abstract* — In the history of Structural Engineering, the methods available for numerical simulation include Finite Element Method (FEM), Finite Difference method (FDM) and Finite Volume method (FVM). In these numerical methods the process of grid formation for irregular geometry requires tedious mathematical transformation; and the treatment of deformation in meshing requires rezoning which is tedious & time consuming. Hence the grid based methods cannot be used to solve problems with large discontinuities and deformations. So meshfree particle method is chosen to overcome this limitation. Mesh free particle method is a particle based mesh free approach where the computational frames in MPM are moving particles in space. There is no necessity of predefined connectivity between the particles. This particle method based on the concepts of Lagrangian theory, whereas the approximation of any desired function in MPM method is carried out in two stages namely kernel and particle approximation. The resulting function from the particle approximation is numerically simulated to obtain the required functional output. Hence MPM serves as a versatile method to handle problems with highly irregular and even dynamic geometry with large deformations. Owing to these advantages, this method has been largely applied to solve different problems in Structural engineering. Typical examples include issues with free surface, deformable frame, moving interface and large deformation. This paper presents an overview on the concepts and benefits of MPM method, its recent developments and the future scope for meshfree particle methods in Structural Engineering applications.**

Keywords:* *Applications, MPM, Scope, Structural Engineering.

I. INTRODUCTION

A. Traditional Numerical Method

Numerical simulation is one of the most commonly used methods to solve complex problems in this technical world. It is a promising method to solve complicated physical problems and offers a new perspective for the discovery of new ideas. Grid based numerical methods such as Finite Element method (FEM), Finite Volume Method (FVM), Finite Difference Method (FDM) are largely being applied in the field of computational solid and fluid dynamics (CSD and CFD). These numerical methods are largely useful in solving the real time physical phenomenon[1]. For many decades FDM has been widely used to solve simple geometrical problem whereas FEM has been used to solve solid mechanics problems. The principle in grid based methods is to divide continues domain by the process of discretization. This discretization results in numerous smaller sub domains by which each domain is inter connected by means of a node. Before the function approximation is carried out in these grid based methods, it is essential to define the inter-relationship between the elements. Based on the inter-relationship of the elements, the governing partial differential equation of the required function is converted into algebraic equations. The field variables are calculated at the nodal points only. Till now, in the history of numerical simulation, grid based methods are the dominant methods to solve complicated problems in engineering and technology.

Though the grid based methods are the dominant methods, their applications are limited to complicated problems due to the use of mesh, which should guarantee the similarity between the physical and numerical compatibility conditions[1]. This leads to the limitations to solve issues with free surface and discontinuous boundary. And also in the grid based techniques, the generation of standard grid has become tedious and also uneconomical. These methods are mostly preferred to solve problems involving discrete physical particles only.

In Eulerian method, the construction of a grid requires complex mathematical transformations and hence it is a difficult task. The process of determining the position of in-homogeneity within the boundary is also a hard deal to perform. Hence the Euler's technique is not suited for particulate flow problems[1]. In Lagrangian method, mesh generation consumes much time and effort. Till now, it is challenge to solve problems with large deformation using Lagrangian theory since it requires special techniques like rezoning. The use of grid based methods to solve problems with continuous domain is often not recommended.

The limitations of grid based methods shows a great influence during solving hydrodynamic problems such as explosion and high velocity impact as there exists large deformation and in-homogeneities in these problems.

B. Mesh Free Method

During the past generations, MPM has been a focused research area for efficient numerical simulation since it provides stable numerical solutions for the differential equation. Based on the mathematical model used, MPM can be classified as probabilistic or deterministic MPM. The deterministic MPM handles the equations of physical law with the initial boundary conditions by which the particle functions can be evaluated at later stages

The benefits of MPM over mesh methods can be summarized as follows[2].

- Lack of connectivity between particles enables the handling of large deformations problems much easier when compared with grid based methods.
- Initial discretization is alone sufficient for particles inside the complex geometry.
- Particle refinement is relatively easier than mesh refinement.
- Motion of particles can be traced at every time step.
- Identification of deformable boundaries is not a difficult task.
- Time history of particles can also be easily determined.
- The solution to the solid mechanics problem involves weakened weak formulation instead of weak formulation in FEM. These problems find its application in Structural Engineering such as shock wave, dam break behavior, etc[3].

II. MPM APPROXIMATION TECHNIQUES

For the numerical simulation of any function, some salient points have to be noted in MPM which are listed below.

- The domain of the geometry is assumed to consist of finite number of particles which are free to move in the space within the domain without any pre defined connectivity between the particles. This concept is called mesh free.
- Integral representation method is used for the real time approximation of the function which is termed as kernel approximation.
- The integration in the kernel approximation is replaced with summations over all the respective values at the neighboring particles in a local domain called the support domain. This condition is called compact condition and this approximation is termed as particle approximation.
- The current distribution of particles can be identified at every time step and it is the local distribution of particle at any time‘t’.
- The particle approximations convert the partial differential equations to ordinary differential equations in discretized form in Lagrangian form.
- The resulting ordinary differential equations are solved using any numerical simulation software to obtain the dynamic properties of field variables[3].

A. *Kernel approximation*

Concept of kernel approximation is based on the property of Dirac delta function.[1] In kernel, the real time function of a particle is expressed as follows.

$$f(x) = \int_{\Omega} f(x')\delta(x-x')dx'$$

Here f is the real time function and δ (x-x’) is the Dirac delta function given by,

$$\delta(x-x') = \begin{cases} 1 & x = x' \\ 0 & x \neq x' \end{cases}$$

If the delta function kernel is replaced by a smoothing function W (x-x’, h) the real time function can be expressed in terms of its elementary function and its smoothing function in the continuum domain.

$$f(x) = \int_{\Omega} f(x')W(x-x',h)dx'$$

Here f(x) is the real time function, f (x’) is the elementary function and W (x-x’, h) is the smoothing function. Smoothing function is of utmost importance in MPM since it not only determines the pattern of interpolation but also deals with the influence area of the particle. The smoothing function provides the weight for each particle within the influence area. This weight of particles will be used in the simulation of interaction of particles. The choice of smoothing function depends on the degree of accuracy expected in the result. The most commonly used smoothing functions include cubic spline, quadratic, quartic, b- spline, etc. New smoothing functions can also be constructed to obtain the required consistency of the function. For any type of smoothing function, the salient properties to be satisfied by the function are listed below.

Few conditions for a smoothing function

- Normalization condition
  $$\int_{\Omega} W(x-x',h)dx' = 1$$
  This condition is also called unity condition because the integration of a smoothing function is equal to one.
- Delta function property
  $$\lim_{h\to 0} W(x-x',h) = \delta(x-x')$$
- Compact condition
  $$W(x-x',h) = 0 \; when \; |x-x'| > kh$$
  This condition expresses that for particles outside the influence domain, the smoothing function is zero. ‘*K*’ is a constant related to smoothing function for particle x and it defines the effective area of a smoothing function.
- All smoothing functions should be decreasing monotonically.
- The functions must be an even symmetric function.
- The functions must be sufficiently smooth without any oscillations.

B. *Particle approximation*

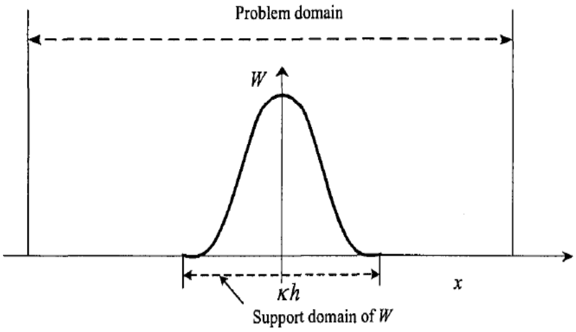
In MPM the entire domain is represented by a finite number of particles which carry its own mass and space. Mass of the particle is represented in terms of its density and volume by the following expression where m_j , ΔV and ρ_j are the mass, volume and density of j^{th} particle (j= 1,2,….N) in which N is the number of particles within the influence domain.

$$m_j = \Delta V_j \rho_j$$

The function obtained from the kernel approximation is expressed as discrete function in the local domain called as support domain.

Fig 1 represents the support domain of the smoothing function W[1].

Fig 2 represents the particle approximation scheme of the particles inside the compact support[1].



.Fig. 1 Support domain of smoothing function W

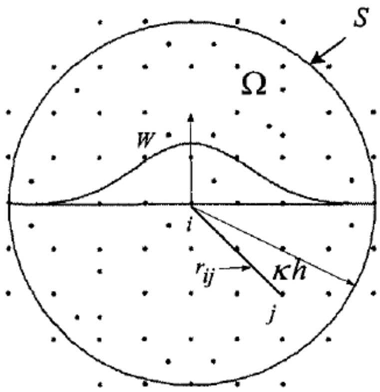


Fig. 2 Particle approximation using particles inside the compact support of the smoothing function W

$$f(x) = \sum_{l=1}^{N} \frac{m'}{\rho'} f(x') W(x - x', h)$$

Here f(x) is the real time function, f (x') is the elementary function and W (x-x', h) is the smoothing function. Hence the particle approximation converts continuous integral of the real function to discretized summations based on set of particles. In this step mass and density of the particles are introduced into the equations.

III. APPLICATIONS OF MPM IN STRUCTURAL ENGINEERING

Initially the concepts of MPM were applied in the field of astrophysics, interstellar collisions, and galaxy theories and also in the evolution of universe. But now this method has been extended to solve problems in structural engineering also. The applications of MPM in structural engineering include the following.

- Dam break problem
- Shock wave problem
- Buckling analysis of plates
- Fracture mechanics
- Vibration in beam problems

A. Dam break problem

In reality the dam break causes a rapidly varied unsteady flow of water in the downstream. This flow of water is an important discussion in structural engineering due to its damages on human society which is caused due to its high velocity and pressure. The causes of dam break can be summarized as follows.

- Overtopping caused by floods.
- Material failure in the structure.
- Failure of foundation.
- Settlement of embankment dams.
- Piping failure.
- Inadequate maintenance.

In the history, the numerical modeling of dam break has been carried out using Eulerian method which is more difficult to generate a high resolution waves[4]. These surface flows can be simulated using MPM since it is an appealing method to solve complicated geometries with large deformation.

B. Shock wave problem

The explosion in general can be classified into physical, nuclear and chemical explosion. Failure of gas cylinder and eruption of volcano are examples of physical explosions. The energy released during formation of nuclei is called nuclear explosion. In chemical explosion, most of the explosives are condensed. The process of reaction of an explosive is called detonation and it produces a blast wave.

The explosive can be classified as primary and secondary explosive based on sensitivity of ignition. Lead azide, mercury fulminate are examples of primary explosives. Tri nitro toluene, ammonium nitrate fuel oxide are examples of secondary explosives. TNT is the standard explosive used.

One of the important properties of blast wave is its short duration and high magnitude. The explosion occurs with a sudden release of energy. Then the energy moves outwards to compress the ambient air. Then the compressed air moves forward with a velocity front. The wave profile is dependent on type of explosive and distance from the source. As per Friedlander's theory blast wave can be categorized into three zones.

- Shock wave (Rarefaction wave)
- Slower subsonic front
- Negative pressure wave

Impact of shock wave on structures is the main cause of structural damage leading to the death and injuries to humans[5]. One of the important step to prevent the structural damage is to estimate the design loadings precisely. MPM serves as a promising method to accurately determine the design parameters in spite of large deformation. The design parameters may include pressure, velocity, energy and density of the air. These parameters are accurately estimated using Mesh Free Particle method to determine the design loadings on the structure.

C. Buckling analysis of plates

Owing to the high stiffness to weight ratio, stiffened plate structures are widely used in structures. The stability of these structures controls the optimum design of structures by which the designer can choose the plate thickness and stiffener dimensions. Numerical tables are available in literatures to calculate the buckling load for rectangular plate stiffened by longitudinal and transverse ribs. Post buckling analysis of stiffened panels can be done using a semi analytical approach. Buckling analysis of stiffened plates are also studied using Finite element models using iso-parametric bending element and four noded rectangular elements. The development of material and geometric non linear spline strip method is also used to analyze the panels subjected to axial compression[6]. Among the numerous methods for the analysis of plates, the most common method used is the FEM. However the existence of difficulties such as distortion of elements causes erroneous result. To avoid this difficulty in meshing, meshless methods are preferred where the displacement approximation is defined based on set of particles within the influence domain. The derivatives of the function are approximated without differentiating the basis function. Taylor's series is used to relate each particle with the neighboring particle. Wider class of kernel functions can be used based on the accuracy of results required. In FEM, Modification of stiffener location leads to re meshing of the entire structure and stiffener. However in MPM, it is avoided.

D. Fracture mechanics

Since the grid based methods are not applicable to problems with large discontinuity, problems of fracture mechanics can be solved easily using MPM through which mesh distortion can be avoided. The mesh free methods are more versatile, adaptive and robust for fracture mechanics problems. MPM provides most accurate and stable numerical solutions for integral equations or partial differential equations with all possible boundary conditions without the use of mesh[7]. The provision of arbitrarily distributed nodes facilitates the increased accuracy in the solution of fracture mechanics problems. The solution of MPM starts with the generation of nodes or triangulation. Shape functions of any desired order continuity are generated through these nodes by integral representation method or series representation method or differential representation method. Then the essential boundary conditions or support conditions are generated through orthogonal transformation technique or direct interpolation method. The solution for stress and strain can be obtained through the solution of displacement. Post processing is not required for smooth derivatives of the unknown parameters.

E. Vibration in beam problems

In beam problems, the deflection and slope are the primary parameters. The deflection parameter can be interpolated with Radial Basis function using MPM technique. The interpolation of parameters is done at any particle within the problem domain. Shape functions for deflection and slope can be constructed at each particle. Then interaction between the particles is simulated using kernel and particle approximations within the compact support. The functions are numerically simulated to obtain the primary parameters of the beam problem. Since the deflection and slope are inter related to each other slope at any location of the beam can be interpolated from the function of deflection[8].

In dynamic analysis of the structure, vibration plays a vital role in the response of the structure. In vibration engineering, the modal parameters have some important information in the design and have a influence on forced response behavior of the structure. The methods available in the literature to solve the vibration problems include experimental and analytical methods. Due to the mathematical complications, high requirements of modeling of practical problems in real time, the analytical and experimental method are not suitable to solve these vibration problems. Since the particles in meshfree methods are randomly distributed without any interconnectivity between the nodes, MPM helps us to solve this vibration problem without any complex mathematical transformations.

IV. CONCLUSION

Particle methods are the upcoming generation of tools for solving numerical simulation. MPM is one of the successful methods for solving numerical problems. In spite of huge success, some constraints of MPM limit its application in solving certain problems. In shock wave problems, development of particle re-distribution helps in increased accuracy in the particle approximation. In the present work, salient points on traditional numerical method, mesh free methods, mesh free approximation techniques including kernel and particle approximations were discussed and the applications of MPM in Dam break problem, Shock wave problem, buckling analysis of plates, fracture mechanics, vibration in beam problems are presented. These various applications show the potential of MPM to be further extended in the field of structural engineering.

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